# A Continuous-Time Electricity Market Model and Its Application to Evaluation of Effects of Climatic Change*

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**Running title:** Electricity Market Model

### Abstract

This paper sets out a tractable continuous-time model of an electricity market that properly incorporates forward-looking generation decision-making in an environment of multiple periods, gas and hydro generation, uncertain inflows, and water storage options. The model is used to examine the effect of changes in water reservoir inflow characteristics, resulting from climatic change, on welfare, electricity output and prices, and the value of additional reservoir investment. It reports outcomes under the polar market structures of competition and monopoly. Calibrated to the New Zealand Electricity Market, it suggests that reductions (increases) in average inflows yield decreases (increases) in welfare, whereas changed volatility of inflows have minor welfare effects. The value of additional reservoir investment

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is sensitive to these changes in the distribution of inflows: it varies importantly between producers and consumers even under competition. Welfare is lower under monopoly, but so is volatility of price and output that is not measured in welfare. The implications of a carbon tax are reported.

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# 1 Introduction

We present a model of an electricity market and use it to assess the effect of climate change on market performance. There is increasing demand for high quality electricity,[1] augmented by the evolving penetration of digital products in modern economies. While there are many possibilities for the evolution of alternative sources of electricity that consume fewer non-renewable natural resources than at present, these technologies are unlikely to have a material impact in the medium term. Economically efficient production and transmission of electricity using existing technologies therefore remains important, where economic efficiency incorporates all facets of competing resource use. Our model incorporates stochastic reservoir inflows and it takes existing technology and the structure of the electricity market as given while exploring how variations in the inflow process and a carbon tax affect market performance.

Electricity markets are dynamic in the short and long term. Our model incorporates forward-looking intertemporal decision making that properly reflects fuel uncertainty and its management by storage. It explicitly incorporates intertemporal linkages that are present in electricity generation even in the short run. These characteristics combine to render the model suitable for exploring the implications for electricity market performance of potential climatic changes and policy instruments. We calibrate the model to the New Zealand Electricity Market (NZEM) that is an energy-only market in which hydro-generation supplies some 55–65 percent of generation capacity.[2] The capacity of the storage lakes is low and inflows to these reservoirs are volatile. We use this model to explore the effects on electricity prices and production of variations in the stochastic properties of inflows that might occur under climate change, and of policy changes such as a carbon tax. Because our model is tractable and admits different market structures we report outcomes under competition and monopoly.

Much economic modelling of electricity markets focusses on assessing market power. Alternative, popular approaches are termed strategic offering and direct analysis and compare actual price and supply outcomes with those arising in a *static* full information electricity market under

[1] That is, electricity with specific characteristics and continuous availability.

[2] See Evans and Meade (2005, Ch.3).

perfect, and Cournot, competition. Strategic offering analysis compares the actual offers of individual generators with estimates of the operational marginal cost of turning fuel into electricity. Wolfram (1998) applies this approach to the UK electricity market and Joskow and Kahn (2002) apply a similar approach to the California electricity market with respect to the summer of 2000. Wolak (2001, 2003) justifies the use of revealed bid information to estimate Lerner indices in a static model. In a variation, direct analysis constructs static supply curves for the entire market. It was applied by Borenstein, Bushnell, and Wolak (2002) and Joskow and Kahn (2002) in studying the California market.[3] These approaches have not used forward-looking supply curves that reflect resource availability uncertainties and storage: rather they treat suppliers as myopically reacting to marginal cost that is public information. These uncertainties are intrinsic to de-regulated electricity markets, and Evans and Guthrie (2009) show that they materially affect generator behavior and consequently market outcomes and the interpretation of electricity market data. Twomey et al. (2005, p. 23) acknowledge opportunity cost measurement issues in estimating marginal cost but do not suggest a solution.[4] Hansen (2009) does take account of this opportunity cost in a model that has two periods, hydropower and no thermal generation, uncertain inflows in the second period, and no spillage.

Dynamic decision making in electricity markets has been considered in other literatures. Operational research models have been constructed to simulate electricity systems and incorporate generator behavior; they are typically in discrete time, complex and not comprehensibly tractable. The discrete-time model of Scott and Read (1996) falls in this literature. Differences from our model include that it does not have period-by-period generator decision making under uncertainty driven by a time dependent stochastic inflows. Other literature implements a Bayesian-Nash equilibrium approach to the operation of electricity markets (for example, Hortescu and Pullar (2008)).[5] This approach is used in "hybrid" models that employ electricity market equilibrium models to explain prices in terms of observable quantities, assume dynamic behavior

[3] Green and Newbery 1992; Wolfram 1999; Bushnell 2005; Bushnell, Mansur, and Saravia 2004 also employ direct analysis in their modeling.

[4] Müsgens (2006) also uses the standard approach and does not mention fuel availability.

[5] See also Espinosa and Riascos (2010).

of state variables, and apply no-arbitrage conditions to calculate derivative prices.[6] While this approach is forward looking it does not lend itself to calibration to natural resource characteristics—such as uncertain sequences of fuel availabilities and its management by storage—that will be informative about the effect of changes on electricity market performance; or permit study of the mix of generators with different fuels.

In our application we calibrate our model to the New Zealand electricity market. We do not distinguish between climatic cycles of a stationary environment (Brönniman et al. (2008)) and irreversible climate change (Stern, 2006), but explore the effect of potential climatic change on electricity price and quantity, the composition of fuels used to generate electricity, and the role of storage, under different market structures. The particular scenarios we consider are changes in the average, predictability and volatility of flows of water to hydro reservoirs. We also consider the implications of different reservoir sizes and the availability and cost of gas. The latter will include the possibility of a carbon tax. We present values of stored water as part of our findings. Possibilities for New Zealand climate change are suggested by the Ministry for the Environment include increased volatility of weather.[7] This seems to be a widespread prognostication (see Katz and Brown (1992)), and we address it directly.We find that if climate change reduces the long-run level of inflows to hydro storage lakes then it will reduce welfare significantly. The effect of increases is symmetric, with increased inflows producing significant welfare increases. If climate change increases the volatility of these inflows, or decreases their rate of mean reversion, then it will produce more volatile outcomes, but the effect on welfare will be small. Monopoly reduces welfare measured *ex post* in each trading period, uses a different mix of generation, and significantly reduces price volatility for the same quantum of storage capacity. A carbon tax has predictable effects in that it induces the substitution of hydro for gas generation, and increases the volatility of prices and total surplus. The carbon tax reduces the total surplus of the electricity market. The value of a larger reservoir is sensitive to the different climate change

[6] See, for example, Skantze et al. (2000) and Lyle and Elliott (2009).

[7] See Ministry for the Environment at http://www.mfe.govt.nz/publications/climate/climate-change-effect-impacts-assessments-may08/html/page5.html (accessed 15 April 2010).

scenarios. We show that an expansion in the reservoir would be of benefit where inflows become more volatile, but of negligible benefit when average inflows fall. In general there may be a conflict between consumers and generators because in some settings reservoir expansion induces conflict between consumer and producer surplus, even when it raises total surplus.

In Section 2 we set out the model and assess the market outcomes it produces under alternative market structures. In Section 3 we modify stochastic resource availability in ways that potentially may occur as a result of climate change and assess how these affect market performance. We draw conclusions in Section 4.

# 2 An electricity market model

## 2.1 The structure

The model we specify has gas and hydro generators that sell into an electricity spot market, and consumers that purchase directly from that market. Although it treats both generators as being held by one owner, decisions are taken such that market outcomes are those a central planner would choose in seeking a social welfare maximum. It will mimic outcomes of a competitive electricity market in which each generator holds a gas and hydro generator.[8] We contrast these outcomes with those of monopoly ownership of all generators.

The model will be calibrated to the New Zealand Electricity Market (NZEM). It has a general form that may be calibrated to other markets. In this specification it presumes a network with three nodes: one each for the gas and hydro generators and one for consumers. The auction and dispatch process for NZEM produces prices and quantities for each half hour. Our model is cast in continuous time, enabling it to closely mimic the real time nature of electricity markets.[9]

[8] We assume an equivalence between the social planner's decisions and perfect competition (Scheinkman and Schechtman, 1983).

[9] A key feature of electricity markets is that, because storage of electricity (in contrast to fuel) is not cost effective, dispatch of generation is managed to meet demand at each instant of time (Stoft, 2002, ch. 1). Prices are determined almost in real time—NZEM has 5 minute pricing (Evans and Meade, 2005), a time weighted average of which produces a price for the period. The 5-minute prices are indicator prices for market participants;

The hydro generator's fuel supply is limited by the availability of stored water and river flows (inflows to the hydro lakes). The inflow at time $t$, measured in terawatt hours per year (TWh/y), is denoted $y_t$ and evolves according to the diffusion process[10]

$$dy_t = \eta(\mu - y_t)dt + \sigma\sqrt{y_t}d\xi_t,$$

where $d\xi_t$ is independently and normally distributed with mean zero and variance $dt$. It follows that, conditional on $y_t$, the rate of inflow at date $t + dt$ has expected value $y_t + \eta(\mu - y_t)dt$ and standard deviation $\sigma\sqrt{y_t dt}$. Inflows are non-negative in this process and the unconditional (time independent) mean and standard deviation of inflows in TWh/y equal $\mu$ and $\sigma\sqrt{\mu/2\eta}$ respectively. The parameter $\eta$ determines the speed with which departures of inflows from the unconditional mean drift back to the mean. As $\eta$ approaches zero the inflow process approaches a random walk without drift, rendering historical inflows no assistance in inflow prediction. In contrast, as $\eta$ approaches infinity the inflow process becomes well approximated by a constant long-run level plus a stationary disturbance term, and thus much more predictable.

We assume that the lake has the capacity to hold water capable of producing $\bar{s}$ TWh of electricity. At date $t$, the storage lake contains water capable of producing $s_t$ TWh of electricity and hydro generators are producing electricity at a rate of $z_t$ TWh/y. Hydro generation is subject to the constraint of plant capacity, $\bar{z}$, so that hydro electricity production satisfies $0 \leq z_t \leq \bar{z}$. It is also subject to the availability of water. Whenever the lake has some spare capacity, the lake level evolves according to $ds_t = (y_t - z_t)dt$. If the lake is empty then output is constrained by $z_t \leq y_t$, so that $ds_t \geq 0$. If the lake is full then the lake level evolves according to $ds_t = \min\{0, y_t - z_t\}dt$; that is, electricity is generated at rate $z_t$ and any inflow in excess of this amount is not used in storage or hydro generation but rather is spilled from the reservoir.

Gas generators are assumed to have increasing marginal cost. If gas generators are producing electricity at a rate of $m_t$ TWh/y at date $t$ then the total cost of generation is $c(m_t) = bm_t^2$.

the transaction prices are calculated after the trading period is closed. Thus, continuous time models provide a good approximation to the operation of these markets. Our model does not address other features of electricity markets, such as seasonality, except insofar as these are reflected in the timing and amounts of inflows.

[10]This stochastic process is used by Cox et al. (1985) to model stationary, non-negative interest rates.

Increasing marginal cost reflects the distribution of efficient thermal plant. Older, or more fuel flexible, plant that are less efficient in converting gas to electricity will have higher marginal costs than will modern plants. A constant term could be included in the cost function to represent the role of must run, or base load, generation, but it would not affect generation decisions. Electricity generated by gas satisfies the constraint $0 \leq m_t \leq \bar{m}$, where $\bar{m}$ is the maximum capacity of the thermal generation plant.

The hydro and gas generators are physically separated from consumers, so that some electricity is lost during the transmission process. Of each unit of electricity generated by the hydro plant, only $k_1$ units are available to consumers; the residual is lost in transmission. Similarly, of each unit of electricity generated by the gas plant, only $k_2$ units are available to consumers. We assume that hydro generators are located at a transmission network node further from consumers than are the gas generators, so that hydro generation experiences greater transmission losses than gas generation: $k_1 < k_2 < 1$.[11]

Consumers reside at the third node of the network. Their demand is captured by the inverse demand function $p_t = \varphi(k_1 z_t + k_2 m_t)$ where $\varphi' < 0$ and the price is measured in dollars per megawatt hour ($/MWh) or, equivalently, millions of dollars per TWh.

## 2.2 Solving the model

The social planner's objective is to maximize the expected present value of the total surplus produced by the electricity market, which looking forward from $t = 0$ is

$$W(s_0, y_0) = E_0 \left[ \int_0^\infty e^{-rt} TS(z(s_t, y_t), m(s_t, y_t) : y_t) dt \right],$$

where

$$TS(z_t, m_t : y_t) = \int_0^{k_1 z_t + k_2 m_t} \varphi(q) dq - c(m_t)$$

[11] Gas-fired generators typically have an option that hydro-generators lack; the option to transmit fuel (gas) and generate in the vicinity of consumers.

and $r$ is the real discount rate.[12] The social planner's objective function has generation expressed in terms of the state of the market (that is, storage and inflows) and thus implies optimal generation policies that depend on storage and inflows. Total surplus does not include costs associated with hydro generation. It assumes that the variable cost of the hydro generation is zero, and that reservoirs and their costs are fixed over time. The presence of the storage option provided by reservoirs will affect the present value of expected surplus by enabling hydro generation to be shifted between time periods. Depending on inflows and generation capacity it will also provide more useable water for generation in aggregate.

The maximum of the social planner's forward-looking intertemporal objective function must satisfy the Hamilton-Jacobi-Bellman equation

$$0 = \max_{z_t, m_t} \; (y_t - z_t)\frac{\partial W}{\partial s} + \nu \frac{\partial W}{\partial y} + \frac{1}{2}\phi \frac{\partial^2 W}{\partial y^2} - rW + TS(z_t, m_t : y_t) \tag{1}$$

subject to the generation constraints $0 \le z_t \le \bar{z}$, $0 \le m_t \le \bar{m}$ and, if and only if $s_t = 0$, $z_t \le y_t$. Neglecting the elements of (1) that are not functions of $(z_t, m_t)$, the level of generation will be such that

$$TS(z_t, m_t : y_t) - z_t \frac{\partial W}{\partial s} = \int_0^{k_1 z_t + k_2 m_t} \varphi(q)dq - c(m_t) - h_t z_t$$

is a maximum, where $h_t \equiv \frac{\partial W}{\partial s}|_{(s_t, y_t)}$ is the impact on the expected present value of future welfare resulting from an increment in stored water and thus is the shadow price of stored water. The first-order conditions for an interior solution are

$$\varphi(k_1 z_t^* + k_2 m_t^*) = \frac{h_t}{k_1}, \tag{2}$$

$$\varphi(k_1 z_t^* + k_2 m_t^*) = \frac{c'(m_t^*)}{k_2}. \tag{3}$$

For each of optimal hydro and gas generation the consumer price equals marginal cost (inclusive of transmission losses). In the case of hydro generation, marginal cost includes the opportunity cost of using stored water at time $t$ rather than leaving it in storage and using it for generation at some later date. The solution to the social planner's problem must satisfy (1)–(3) simultaneously.

[12] We assume that expectations are based on risk-neutral probabilities and hence that $r$ is the risk-free interest rate (see Lengwiler (2004, pp. 44–45)).

However, it can be partitioned into a static and a dynamic problem with the link provided by the shadow price of water, $h_t$. The shadow price can be taken as exogenous when choosing generation levels at time $t$ (via (2) and (3)), but $W$, and hence $h_t$, must satisfy (1) and the dynamic constraints

$$ds_t = \begin{cases} (y_t - z_t)dt, & \text{if } s_t < \bar{s}, \\ \min\{0, y_t - z_t\}dt, & \text{if } s_t = \bar{s}. \end{cases} \tag{4}$$

It influences generation at $t$ by means of (2) and (3). We use this partition to solve the model.

First an exhaustive set of feasible generation policies are obtained by solving (2) and (3) subject to the generation constraints for a grid of possible feasible values for $y_t$ and $h_t$.[13] This first stage yields a set of generation policies $\{\hat{z}(y_t, h_t), \hat{m}(y_t, h_t)\}$ that satisfy (2) and (3) and the generation constraints.

In the second stage the generation policy $\{z(y_t, s_t), m(y_t, s_t)\}$ satisfying (1)–(3) and the dynamic constraints is obtained by an iterative process. It uses a grid defined in $(y, s)$ space where $s \in (0 = s_1 < s_2 < \ldots < s_J = \bar{s})$ increases by increment $ds$ and $y \in (y_{\min} = y_1 < y_2 < \ldots < y_K = y_{\max})$ increases by $dy$. The resulting grid has $JK$ pairs of levels of inflows and stored water. The iterative process is then as follows. Starting with $n = 1$:

1. A guess is made of the generation policy at each point $(y, s)$ as $\{z^{(n-1)}(y, s), m^{(n-1)}(y, s)\}$.[14]

2. Then, $W^{(n)}$ is defined by evaluating (1) at these generation levels thus:

$$0 = (y - z^{(n-1)})\frac{\partial W^{(n)}}{\partial s} + \nu\frac{\partial W^{(n)}}{\partial y} + \frac{1}{2}\phi\frac{\partial^2 W^{(n)}}{\partial y^2} - rW^{(n)} + TS(z^{(n-1)}, m^{(n-1)} : y).$$

3. Taking as the shadow price of water $h^{(n)} = \frac{\partial W^{(n)}}{\partial s}$, optimal generation produced at each point $(y, s)$ is given by

$$z^{(n)}(y, s) = \hat{z}(y, h^{(n)}(y, s)) \quad \text{and} \quad m^{(n)}(y, s) = \hat{m}(y, h^{(n)}(y, s)).$$

[13] In this description we use interior solutions. In fact, solving (1)–(3) requires meeting the generation and dynamic constraints. The solution process is altered when these constraints are binding.

[14] The initial guess is a level of generation within the generation and dynamic constraints.

4. The second and third steps are repeated until each of $|m^{(n)} - m^{(n-1)}|$, $|z^{(n)} - z^{(n-1)}|$, $|h^{(n)} - h^{(n-1)}|$, and $|\varphi(k_1 z^{(n)} + k_2 m^{(n)}) - \varphi(k_1 z^{(n-1)} + k_2 m^{(n-1)})|$ is smaller than an arbitrary threshold, which we set to $10^{-6}$.

These policies mean that the optimal choice of generation and storage are available for each $(y, s)$ position on the space defined by the grid.

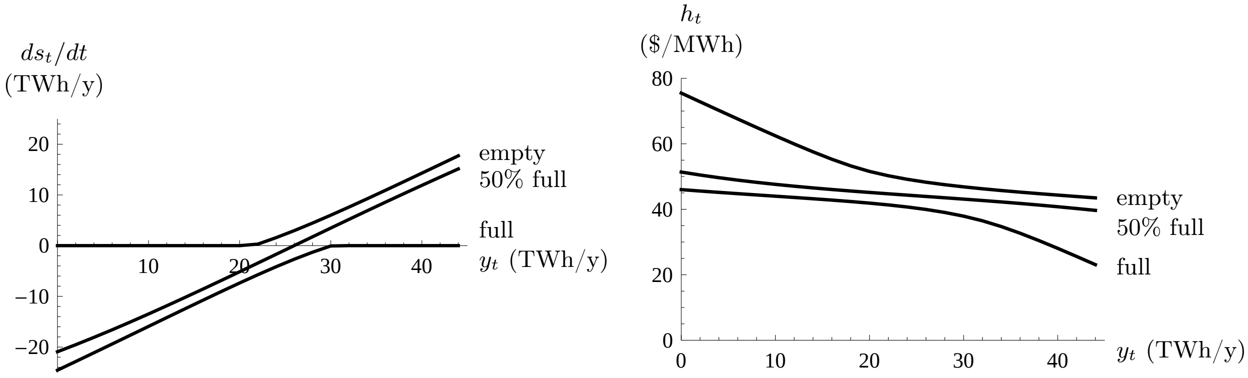
This process yields the social planner's optimal generation policies (that is, $(z_t, m_t)$) for each point on the $(y, s)$ grid.[15] The left-hand graph in Figure 1 illustrates the optimal storage policy for the calibration described in Appendix A: each curve plots the rate of change in the lake level, given in equation (4), as a function of the current rate of inflow ($y_t$) for a different lake level ($s_t$). The lake fills most quickly when inflows are high and the lake is almost empty; it empties most quickly when inflows are low and the lake is almost full. The right-hand graph plots the shadow price of water, $h_t = h(s_t, y_t)$, as a function of inflows for the same three lake levels.

The shadow price of stored water embodies the intertemporal concerns that affect optimal storage policy. It reflects all the characteristics of the electricity market contained in the model and it jointly determines contemporary generation via the first-order conditions (2) and (3). Because it is the expected value of the last unit of stored water looking forward over the foreseeable future, it is affected by the characteristics of the inflow process—its average level, volatility and predictability—as well as the particular structure of the model and generation decisions. In particular, given the inflow characteristics, this expectation will be affected by participants' objective functions, period-by-period volatility in electricity outcomes, and measures of welfare. Thus, the levels of $h_t$ resulting from changes in the electricity market operating environment—for example, inflow characteristics and reservoir size—are complex to predict.

In the case of monopoly exactly the same four steps are applied but using the monopolist's

[15]The iterative solution is obtained for all points on the grid and the solutions for intermediate combinations of $(y, s)$ are obtained by interpolation.

Figure 1: Optimal storage policy and the shadow price of water




first-order conditions

$$\varphi(k_1 z_t^* + k_2 m_t^*) + (k_1 z_t^* + k_2 m_t^*)\varphi'(k_1 z_t^* + k_2 m_t^*) = \frac{h_t}{k_1}$$

$$\varphi(k_1 z_t^* + k_2 m_t^*) + (k_1 z_t^* + k_2 m_t^*)\varphi'(k_1 z_t^* + k_2 m_t^*) = \frac{c'(m_t^*)}{k_2}$$

instead of (2) and (3).

### 2.3 Synopsis of market outcomes

In this section we use the generation policies constructed above to simulate a daily time series of the state of our model of the NZEM for a 30-year period, using the calibration described in Appendix A. We do this to establish properties of the model and lay the base case with which to assess the effect of climate change to inflows.

This long period ensures that the model is subject to a wide range of inflow scenarios, and that the outcomes are negligibly affected by the starting values of $s$ and $y$. Figure 2 depicts the series of simulated daily inflows, which is constructed using the stochastic process in equation (A-1) in the appendix. It contains 10,590 daily inflow observations, with an unconditional mean of 24.54 TWh/y and standard deviation of 6.13 TWh/y. While most of the inflows lie between 15 and 35 TWh/y, there are some periods with very low, and others with very high, inflows. The lowest inflow of 10TWh/y occurred in the 22nd year, whereas a very high inflow of 59TWh/y occurred in the 23rd year. The inflows are serially correlated.

Market outcomes include the model's outputs of daily hydro and gas generation, together

Figure 2: Thirty years of daily inflows ($y_t$)

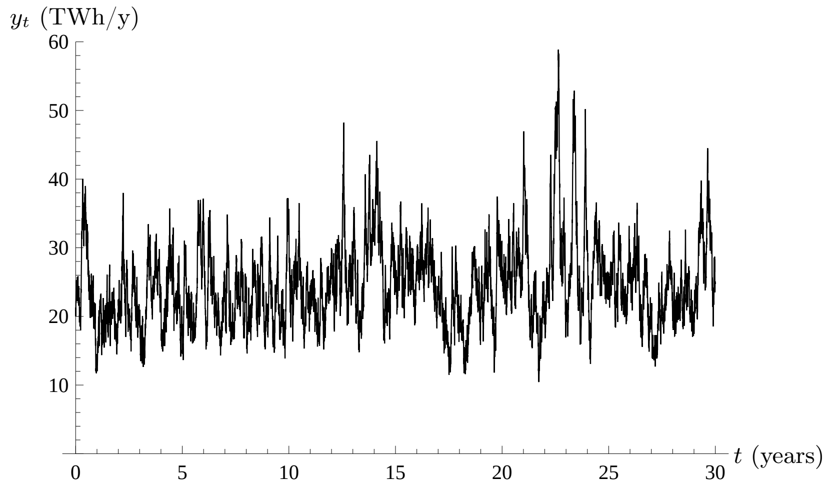
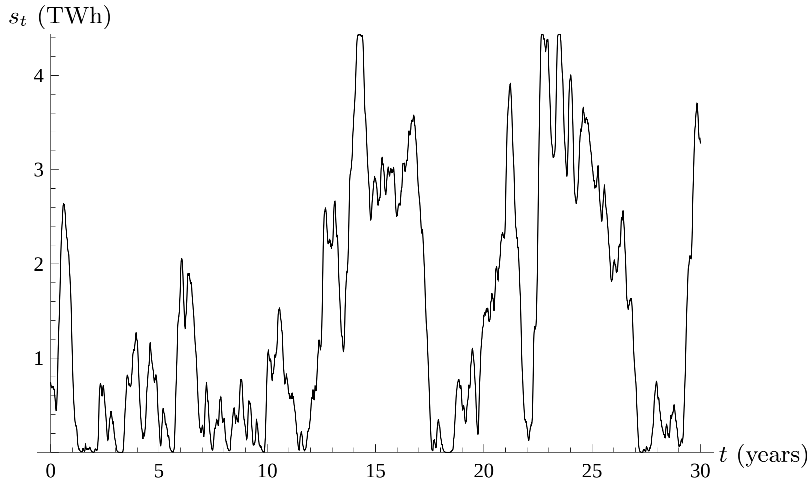



Figure 3: Level of storage ($s_t$) under competition




with the ancillary quantities of storage and spot price. Figure 3 shows the level of stored water during the 30-year period when generation follows the social planner's optimal policies. The average storage level is 1.42 TWh, 32 percent of maximum storage capacity.

The social planner uses and stores lake water continually in applying its generation policy, which is reflected in its higher volatility of inflows (6.13 TWh/y) than the rate of change in the lake level (standard deviation of 4.05 TWh/y) and in the correlation of 0.85 between inflows and the rate of change in the lake level. The lake is generally partially full. However after periods of sustained low (high) inflows the lake becomes empty (full) and this occurs several times during the 30-year period. The time taken to draw down all the water in the lake depends upon the initial level of storage. For example, at the start of the second year, storage has been low (approximately 0.4TWh). The associated low inflows mean that it is socially worthwhile generating from stored water, even to the extent of emptying the lake. In the periods when the lake is full, relatively high inflows pose the likelihood of spilling, in which case the shadow price of stored water will be zero.

The shadow price of water is shown in Figure 4. It is particularly interesting since it is unobserved in electricity markets and must be indirectly estimated. As (2) and (3) indicate, the shadow price of stored water will be determined by the marginal cost of gas, adjusted for transmission costs where there is an interior solution.[16] The shadow price of water has an average value of \$46.06/MWh over the period with a standard deviation of \$4.61/MWh. It is strongly negatively correlated with inflows (correlation coefficient $-0.78$) and the level of storage (correlation coefficient $-0.83$). These correlations are to be expected, given that $h_t$ is the price of stored water, which will be low when current and/or anticipated future lake levels are high. When the lake is full there is no opportunity cost of using inflow for generation, as is shown by comparison of Figures 2 and 3: the shadow price falls significantly when the lake is full. The drop is particularly sharp in the 23rd year when inflow exceeded hydro generation capacity and excess water had to be spilled resulting in a very low shadow price of water.

The level of hydro generation reflects the shadow price of stored water and is shown by the top curve in Figure 5; the bottom curve shows the level of gas generation. The average annual level of hydro generation is 24.45 TWh, which accounts for 60 percent of total generation. This is not far distant from the 56 percent of hydro generation in NZEM in 2007. While more hydro is used when the shadow price of water is low, the fluctuation in annual hydro generation (standard deviation 3.47 TWh/y) is much lower than fluctuations in the inflows (standard deviation 6.13 TWh/y) reflecting the intertemporal substitution of hydro fuel (water) between periods. Hydro generation is most connected to inflows when storage is very high—see the high spikes in hydro

[16] The marginal cost of gas will also be measured imperfectly where the value of stored gas departs from the spot price of gas, as explained by Evans and Guthrie (2009). In this case measuring the marginal cost of generation separately from the electricity spot price is problematic.

Figure 4: Shadow price of stored water ($h_t$) under competition

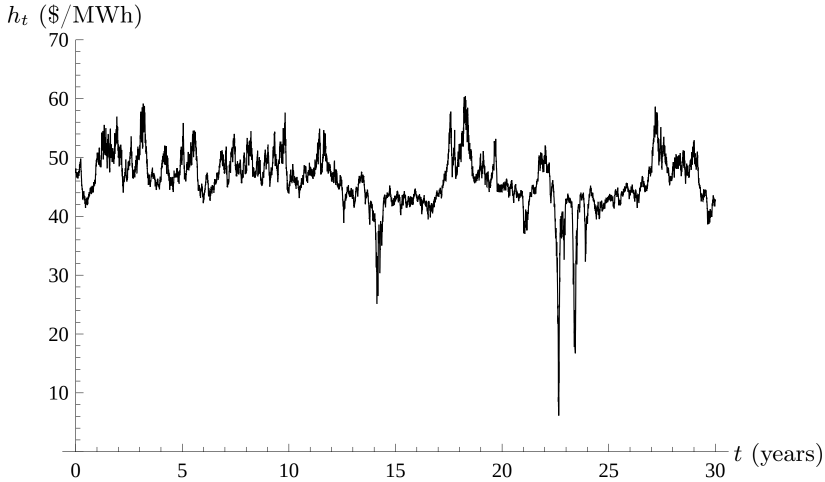
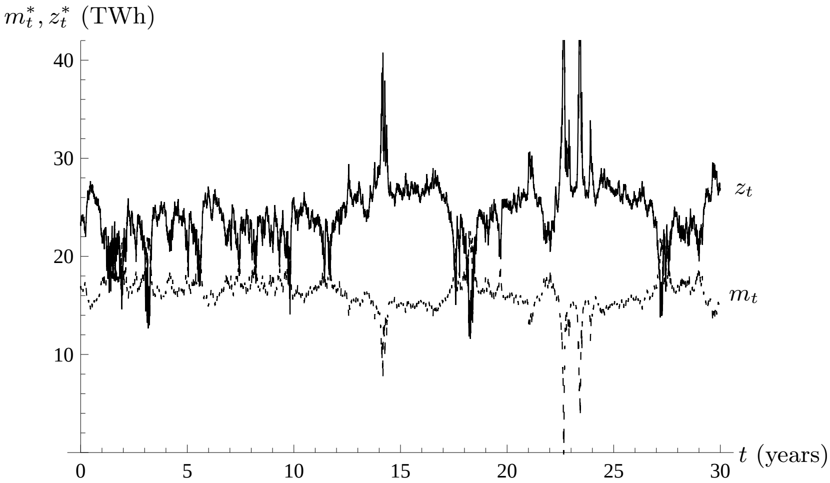


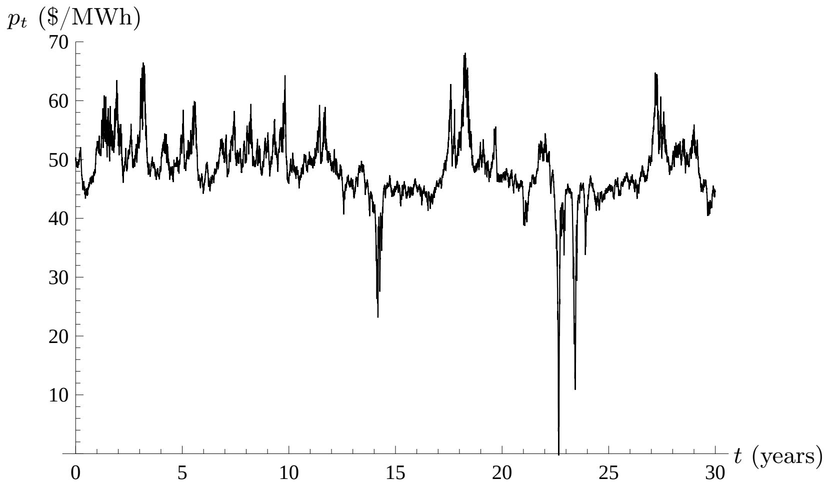

Figure 5: Hydro ($z_t$) and gas ($m_t$) generation under competition




generation in the 15th, 23rd, and 24th years—or very low as revealed at times during the first ten years of the simulation. Following a sustained period of high inflows and associated increases in storage, hydro generation reaches its maximum capacity for two weeks in the middle of the 23rd year. This is the only period in the 30 years where spilling occurs.

Because gas generation transmission costs are lower than the transmission costs of hydro, and the marginal cost of gas equals zero for very small levels of output, gas generation always runs. Over the 30 years, gas generation accounts for 40 percent of total generation at an average of 16.28 TWh/y, and with a smaller volatility (standard deviation 1.80 TWh/y) than hydro. As implied by (2) and (3), gas and hydro generation are at the margin perfect substitutes where

Figure 6: Market price ($p_t$) under competition




there is an interior solution, and because marginal cost is linear and gas generation capacity is not reached in the 30 years, the correlation coefficient between gas and hydro generation is $-1$. However, because gas marginal cost is increasing and the shadow price of water varies, so does total production and gas relative to hydro generation.

The price to consumers in each trading period is given by $p_t = 185 - 3.47(k_1 z_t + k_2 m_t)$ and it is depicted for the 30-year period in Figure 6. Under competition the average market price is \$48.30/MWh. Its standard deviation of \$5.25/MWh produces a coefficient of variation of $CV_p^C = 0.11$ which represents much lower volatility relative to inflows ($CV_y^C = 0.25$) and hydro ($CV_z^C = 0.14$) but the same as that of gas generation ($CV_m^C = 0.11$). Management of storage reduces the volatility of hydro from that of inflows, and the operation of gas generation mitigates the volatility of hydro. The large price falls during the 15th, 23rd, and 24th years coincide with the low shadow price of water that itself coincided with high storage and high inflows.

As mentioned, we include simulated outcomes for monopoly management of the electricity market in order to illustrate the effects of alternative objectives on outcomes, particularly with respect to climate change: monopoly being a different plausible setting for an electricity market.[17] The monopolist manager of the electricity market would generate by means of the process

[17] The NZEM has four moderately large generators and one smaller one. A monopoly outcome could in theory be obtained by collusive strategies, but the New Zealand competition authority has determined on the basis of

described in Section 2.2 with the exception of (2) and (3), which would take the alternative form of marginal revenue equal to the marginal costs of gas and hydro generation. We report only the summary statistics of the monopoly case and do this in the top panel of Table 1 in conjunction with those of the competitive market.

The lower aggregate generation by the monopolist and the consequent higher market price is standard for most monopoly settings. The quantum of these effects will depend upon the calibration. What is most striking about the effect of monopoly management is the much reduced use of gas generation and reduction in volatility of generation and price. Gas generation carries a variable cost that hydro does not and the monopolist, in cutting back on aggregate generation to raise revenue reduces gas generation but not hydro generation. In doing so it is less costly for the monopolist, than the social planner, to manage fluctuations in inflows relatively more by variations in gas generation than hydro generation. This is confirmed by the coefficient of variation of storage, hydro and gas generation under competition ($CV_s^C = 0.90$, $CV_z^C = 0.14$, $CV_m^C = 0.11$) relative to that of monopoly ($CV_s^M = 0.73$, $CV_z^M = 0.09$, $CV_m^M = 0.57$). The lesser generation by the monopolist is reflected in the lower marginal cost of gas and stored water. The generally internal solutions to (2) and (3), and the monopolist's equivalent of these, yield marginal costs of stored water and gas, adjusted for transmission, that are very similar. The difference in the shadow price of stored water between the social planner and monopoly arises in part because of their different objective functions. For the social planner (monopolist), $h_t$ is the increment in the expected present value of welfare (profit) from a unit of stored water and will differ for any given state of the system. The actual magnitude of $h_t$ reflects the difference in objective as well as the effect of the objectives producing lower aggregate generation under monopoly.

The middle panel of Table 1 has the same format as the top panel, but the total storage capacity has been increased from its calibrated value of 4.44 TWh to 5.44 TWh. The effect of the larger reservoir is not to increase the aggregate water available for generation. Indeed, the

a 3-year study by Wolak that there was no affiliated action concerns in this market (New Zealand Commerce Commission, 2009, p. 6).

Table 1: Market outcomes: competition and monopoly compared

| Quantity | Units | Mean | | Standard deviation | |
|---|---|---|---|---|---|
| | | Comp. | Monop. | Comp. | Monop. |
| | | Capacity=4.44 | | | |
| Inflow ($y_t$) | TWh/y | 24.54 | 24.54 | 6.13 | 6.13 |
| Storage ($s_t$) | TWh | 1.42 | 2.04 | 1.28 | 1.49 |
| Price ($p_t$) | $/MWh | 48.30 | 96.36 | 5.35 | 2.19 |
| Hydro gen. ($z_t$) | TWh/y | 24.45 | 24.04 | 3.47 | 2.18 |
| Gas gen. ($m_t$) | TWh/y | 16.28 | 2.60 | 1.80 | 1.48 |
| MC of hydro ($h_t$) | $/MWh | 46.06 | 7.34 | 4.61 | 3.89 |
| MC of gas ($c'(m_t)$) | $/MWh | 47.53 | 7.60 | 5.27 | 4.31 |
| Welfare flow ($TS_t - c(m_t)$) | million $/y | 4200 | 3580 | 152 | 76 |
| Profit flow ($PS_t - c(m_t)$) | million $/y | 1503 | 2447 | 80 | 22 |
| | | Capacity=5.44 | | | |
| Inflow ($y_t$) | TWh/y | 24.54 | 24.54 | 6.13 | 6.13 |
| Storage ($s_t$) | TWh | 1.66 | 2.44 | 1.54 | 1.80 |
| Price ($p_t$) | $/MWh | 48.31 | 96.31 | 4.90 | 2.05 |
| Hydro gen. ($z_t$) | TWh/y | 24.45 | 24.09 | 3.18 | 2.04 |
| Gas gen. ($m_t$) | TWh/y | 16.28 | 2.57 | 1.65 | 1.38 |
| MC of hydro ($h_t$) | $/MWh | 46.05 | 7.26 | 4.26 | 3.67 |
| MC of gas ($c'(m_t)$) | $/MWh | 47.54 | 7.51 | 4.83 | 4.04 |
| Welfare flow ($TS_t - c(m_t)$) | million $/y | 4201 | 3582 | 144 | 70 |
| Profit flow ($PS_t - c(m_t)$) | million $/y | 1505 | 2448 | 61 | 20 |
| Social value of 1TWh of capacity | million $ | 13 | 52 | | |
| Market value of 1TWh of capacity | million $ | 39 | 23 | | |

generation levels are negligibly affected by the extra storage.[18] What the larger reservoir does do is enable some different scheduling of gas and hydro generation with the effect that the average amount stored and its standard deviation increase, whereas generation and price volatility fall. This change in volatility is endogenous in that it is the result of the social planner and the monopolist optimally responding to the larger reservoir: it differentially affects consumer and producer surplus and affects overall welfare.[19,20]

The two rows in the bottom panel of Table 1 report the value to the social planner and the electricity generation industry, respectively, of one additional TWh of storage capacity (that is, approximately 23% of current capacity). The entries in the first of these two rows are $W(0, \mu|\bar{s} = 5.44) - W(0, \mu|\bar{s} = 4.44)$, being the increment in the expected present value of the total surplus produced by the electricity market when the lake is initially empty and the inflow is at its long-run level, and when total storage capacity is increased to 5.44 TWh from 4.44 TWh. The second of these rows depicts the change in the expected present value of generator profits (producer surplus) arising from the same change in reservoir capacity. The only difference between the two columns in this section of the table is that in the first one the social planner's optimal generation policy is followed whereas in the second one the monopolist's optimal policy is followed. They reveal that under competition a larger reservoir produces a significant increase in expected producer surplus but at the expense of consumer surplus: the net effect being a relatively small increase in total welfare. For monopoly, both consumer and producer surplus increase.

[18]The simulation result that negligible spilling occurs during the 30 years suggests maximum storage is not a limitation on aggregate water availability for the calibrated model.

[19]Given the linear inverse demand function $p(q) = a - cq$, consumer and producer surplus are $CS(z, m) = 0.5c(k_1 z + k_2 m)^2$ and $PS(z, m) = a(k_1 z + k_2 m) - c(k_1 z + k_2 m)^2 - bm^2$. Since they are respectively convex and concave, the magnitude of expected consumer and producer surpluses may well move in different directions with changes in the volatility of generation.

[20]The calculations assume that price volatility has no direct effect on the surplus flows. However, if this volatility affected decisions outside the model—such as transmission and generation related investments—the comparison of total surplus as shown in Table 1 would be mis-measured.

## 3 Effect of climatic change

The key climatic variables we investigate are the determinants of the distribution of river flows (and hence reservoir inflows). We also consider prices for gas that would reflect carbon taxes.

### 3.1 Inflows

Climate change over time may affect various aspects of inflows. We consider the effect of 30 percent changes in each of the long-run mean ($\mu$), the volatility ($\sigma$), and the rate of mean reversion ($\eta$), in each case keeping the other parameters at their calibrated levels. Because the qualitative effects of these changes can generally be inferred from two points, we limit consideration to a 30 percent reduction in the unconditional mean, a 30 percent increase in volatility, and a 30 percent decrease in the rate of mean reversion.

The effects of the reduction in mean inflows on market outcomes are indicated in Table 2. Comparison with Table 1 shows that the reduction in inflows decreases total generation, induces substitution of gas for hydro generation, reduces welfare, and substantially reduces the value of additional storage capacity. These results are unsurprising, for the reduction in average inflows lowers the volume of generation fuel over the period. The proportionate increased use of gas by the monopolist is more than that of the competitive market, which is a consequence of its much lower use of gas under the standard calibration. The price does not increase under monopoly proportionately to that of competition and this is reflected in monopoly's proportionately lower reduction in welfare from the reduced inflows. The shadow price of stored fuel rises on average for both the social planner and the monopolist; although they value less an increment in reservoir storage. This result reflects the lesser role of storage induced by the reduced demand for the option to shift hydro generation between time periods and lower likelihood of unused spilled water. In both cases producer surplus expands at the expense of consumer surplus, although total surplus increases.

At a higher conditional volatility of inflows it might be expected that more use is made of the storage facility to smooth generation fuel use over time and hence maintain or increase welfare.

Table 2: Market outcomes at 30% lower average inflows ($\mu|\sigma,\eta$)

| Quantity | Units | Mean | | Standard deviation | |
|---|---|---|---|---|---|
| | | Comp. | Monop. | Comp. | Monop. |
| Inflow ($y_t$) | TWh/y | 17.15 | 17.15 | 5.17 | 5.17 |
| Storage ($s_t$) | TWh | 1.17 | 1.56 | 1.18 | 1.33 |
| Price ($p_t$) | \$/MWh | 59.68 | 103.41 | 4.32 | 2.46 |
| Hydro gen. ($z_t$) | TWh/y | 17.08 | 17.02 | 2.80 | 2.45 |
| Gas gen. ($m_t$) | TWh/y | 20.11 | 7.36 | 1.46 | 1.66 |
| MC of hydro ($h_t$) | \$/MWh | 56.87 | 20.79 | 3.72 | 4.35 |
| MC of gas ($c'(m_t)$) | \$/MWh | 58.72 | 21.48 | 4.25 | 4.85 |
| Welfare flow ($TS_t - c(m_t)$) | million \$/y | 3822 | 3307 | 158 | 107 |
| Profit flow ($PS_t - c(m_t)$) | million \$/y | 1556 | 2347 | 22 | 50 |
| Social value of 1TWh of capacity | million \$ | 4 | 9 | | |
| Market value of 1TWh of capacity | million \$ | 17 | 16 | | |

Since the volume of fuel available over the 30 years is not changed by an increase in $\sigma$, the average price and welfare should not be much affected, but the volatility in these will likely be higher as not all the extra volatility can be smoothed by water storage. Of course, the use of additional gas generation to smooth out the result from fluctuating inflows will raise costs. This is confirmed by comparison of Table 3, which shows the effects of increased inflow volatility, with Table 1. In the case of competition, increased inflow volatility produces a very small substitution of gas for hydro generation, higher storage, and increases the volatility of all market outcomes on a proportionate basis (except storage itself) by at least the increase in water inflow volatility. Average price is virtually unaffected but prices are much more volatile, and the value of additional storage capacity is significantly greater than in the benchmark case. Since volatility of prices does not directly affect the welfare measure, welfare exhibits a small decline that reflects the greater difficulty in substituting generation between time periods relative to the base case. The higher volatility raises the value of an increment to the reservoir because the ability to shift water between time periods, including avoiding spillage situations, is of more utility when inflows are

Table 3: Market outcomes at 30% higher inflow volatility $(\sigma|\eta, \mu)$

| Quantity | Units | Mean | | Standard deviation | |
|---|---|---|---|---|---|
| | | Comp. | Monop. | Comp. | Monop. |
| Inflow ($y_t$) | TWh/y | 24.48 | 24.48 | 8.06 | 8.06 |
| Storage ($s_t$) | TWh | 1.52 | 2.15 | 1.37 | 1.54 |
| Price ($p_t$) | \$/MWh | 48.40 | 96.79 | 7.84 | 2.80 |
| Hydro gen. ($z_t$) | TWh/y | 24.39 | 23.61 | 5.08 | 2.79 |
| Gas gen. ($m_t$) | TWh/y | 16.31 | 2.89 | 2.64 | 1.89 |
| MC of hydro ($h_t$) | \$/MWh | 46.17 | 8.15 | 6.62 | 4.97 |
| MC of gas ($c'(m_t)$) | \$/MWh | 47.63 | 8.44 | 7.71 | 5.51 |
| Welfare flow ($TS_t - c(m_t)$) | million \$/y | 4187 | 3563 | 204 | 101 |
| Profit flow ($PS_t - c(m_t)$) | million \$/y | 1489 | 2441 | 168 | 33 |
| Social value of 1TWh of capacity | million \$ | 33 | 92 | | |
| Market value of 1TWh of capacity | million \$ | 75 | 38 | | |

more volatile. The implications of an increase in $\sigma$ for the monopoly market are very similar, with the exception of the volatility of outcomes. The higher conditional—and hence unconditional—volatility has exacerbated the difference between competition and monopoly in the volatility of input use and outcomes.

A lower inflow mean-reversion parameter, $\eta$, reduces the predictability of the inflows and thus detracts from generators' abilities to forecast particular future benefits of stored water. Concomitantly, it will raise the unconditional volatility of inflows. It is the first of these effects which is important for behavior in the electricity market since period-by-period generation decisions will have more uncertainty about the state of fuel supplies in subsequent periods: that is, the future benefit of fuel. Comparing Table 4, which shows the effects on market outcomes of slower mean reversion in inflows, with Table 1 reveals that the reduced predictability resulting from less rapid mean reversion yields a small increase in average storage, reduced average total surplus and a greater value from additional storage capacity. The changes in the average figures for the competitive market are largely mirrored in monopoly; the relative increase of volatility under

Table 4: Market outcomes at 30% slower inflow-mean reversion ($\eta|\sigma, \mu$)

| Quantity | Units | Mean | | Standard deviation | |
|---|---|---|---|---|---|
| | | Comp. | Monop. | Comp. | Monop. |
| Inflow ($y_t$) | TWh/y | 24.45 | 24.45 | 7.31 | 7.31 |
| Storage ($s_t$) | TWh | 1.49 | 2.10 | 1.42 | 1.62 |
| Price ($p_t$) | \$/MWh | 48.46 | 96.85 | 7.63 | 2.91 |
| Hydro gen. ($z_t$) | TWh/y | 24.35 | 23.56 | 4.95 | 2.90 |
| Gas gen. ($m_t$) | TWh/y | 16.33 | 2.93 | 2.57 | 1.96 |
| MC of hydro ($h_t$) | \$/MWh | 46.17 | 8.24 | 6.57 | 5.16 |
| MC of gas ($c'(m_t)$) | \$/MWh | 47.68 | 8.55 | 7.51 | 5.73 |
| Welfare flow ($TS_t - c(m_t)$) | million \$/y | 4186 | 3561 | 209 | 104 |
| Profit flow ($PS_t - c(m_t)$) | million \$/y | 1491 | 2440 | 135 | 34 |
| Social value of 1TWh of capacity | million \$ | 29 | 82 | | |
| Market value of 1TWh of capacity | million \$ | 68 | 36 | | |

competition that occurred with the increase in $\sigma$ is not present. The reduced forecastability of inflows reduces the utility of inflows *per se* relative to that of stored water or gas. It results in a small increase in the marginal cost of both fuels—stored water and gas—in equilibrium and a small diminution in welfare. The increase in the marginal costs of the monopolist are greater than for the competitive market; and yield a slightly higher reduction in welfare. Both the social planner and monopolist value an additional 1 TWh of reservoir more highly with the 30% reduction in $\eta$, but the social planner's valuation increases more than that of the monopolist in proportionate terms. The additional storage capacity would enable more storage and better predictability to offset the reduction in mean reversion.

Since the unconditional standard deviation is $\sigma\sqrt{\mu/2\eta}$. the decrease in $\eta$ will increase the unconditional variance and the effect of this is indicated in the comparison of the inflow standard deviation of Tables 1 and 4.[21] The form of the unconditional standard deviation implies that the effect of an increase in the speed of reversion to the mean, will be similar qualitatively to an increase in $\sigma$, and this is confirmed by comparison of Tables 3 and 4.

[21] It is the unconditional variance that is being estimated for Table 4 using the 30 years of daily data.

### 3.2 A carbon tax

In this section we consider the application of a carbon tax that has the effect of raising the marginal cost of gas fired plant. New Zealand has not implemented a carbon tax to this point in time, having instead put in place statutes that provide for an emissions trading scheme (ETS)[22] that progressively includes industries, with electricity being among the first. A key feature of an ETS scheme will be the volatility of the price of carbon. We anticipate that it, and the requirement to hold or purchase carbon credits to cover emissions, may produce an input to the electricity sector with the same effects "in-principle" as available water inflows which is beyond the scope of our present model.[23]

We assume that the marginal cost of gas generators is the cost of the gas, and increase it by 24.5 percent being the amount of the tax produced by a carbon tax of \$25/t$CO_2$ produces the market outcomes described in Table 5.[24] The results for a tax of \$50/t$CO_2$ are reported in Appendix B.

The carbon tax raises the marginal cost of gas-fired generation and induces a reduction in it, but little change to hydro generation. The change in the marginal cost of stored water follows the increase in the marginal cost of gas, as theory would predict for an interior solution to our model. In comparison to the unadjusted calibrated model, the reduction in gas use lowers the equilibrium measured increase in the marginal cost of gas from the nominal 24.5 percent to 11.12 percent. It also produces an increase in consumer prices of 11.12 percent, thereby quantifying the sharing of the tax burden between consumers and generators. The more volatile shadow prices of water, gas, and final electricity price suggest that the demand for hydro as a substitute for gas generation is relatively more important than its application to optimising, or smoothing, water

[22] See http://www.climatechange.govt.nz/emissions-trading-scheme/index.html; accessed 16 April 2010.

[23] Our consideration of a carbon tax might be construed as an ETS scheme with no uncertainty and a perfectly elastic supply of carbon permits (credits) at an ad valorem fixed price.

[24] ACIL Consulting (2001, Table 7) reports that an emissions tax of \$10 per tonne CO2 (t/$CO_2$) would induce a cost per Gigajoule (GJ) of gas of \$0.52/GJ. At the 2007 gas price of \$5.34/GJ (reported at www.crownminerals.govt.nz>Home>News>2009, accessed 29 April 2010) this tax produces an increase in gas price of 9.7 per cent. We adjust this figure for tax rates of \$25 and \$50 t/$CO_2$.

Table 5: Market outcomes with a $\$25/tCO_2$ carbon tax

| Quantity | Units | Mean | | Standard deviation | |
|---|---|---|---|---|---|
| | | Comp. | Monop. | Comp. | Monop. |
| Inflow ($y_t$) | TWh/y | 24.54 | 24.54 | 6.13 | 6.13 |
| Storage ($s_t$) | TWh | 1.42 | 2.04 | 1.28 | 1.49 |
| Price ($p_t$) | \$/MWh | 53.98 | 96.98 | 5.98 | 2.54 |
| Hydro gen. ($z_t$) | TWh/y | 24.45 | 24.04 | 3.47 | 2.18 |
| Gas gen. ($m_t$) | TWh/y | 14.61 | 2.42 | 1.62 | 1.37 |
| MC of hydro ($h_t$) | \$/MWh | 51.48 | 8.52 | 5.15 | 4.51 |
| MC of gas ($c'(m_t)$) | \$/MWh | 53.12 | 8.81 | 5.89 | 4.99 |
| Welfare flow ($TS_t - c(m_t)$) | million \$/y | 4114 | 3561 | 170 | 87 |
| Profit flow ($PS_t - c(m_t)$) | million \$/y | 1635 | 2444 | 82 | 25 |
| Social value of 1TWh of capacity | million \$ | 15 | 59 | | |
| Market value of 1TWh of capacity | million \$ | 44 | 26 | | |

use over time. The average and volatility of hydro and generation remain unchanged, but gas generation falls (see Table 5). The welfare effect of the tax is to reduce the total surplus under competition relatively more than under monopoly, reflecting the base case lower use of gas fired generation. The increased volatility of final price resulting from the tax is significant under both market structures, but is not an element of welfare, although the volatility of it too increases with the carbon tax. The value of an additional TWh of capacity under competition and monopoly is not much different from that of the base case. Under competition producers' surplus is higher under the carbon tax than without It might reflect that fact that the non-taxed hydro generators benefit from the carbon tax since the tax raises the price of electricity to consumers.

# 4 Conclusion

We develop a tractable operational model of an electricity market that properly accounts for key characteristics of such markets, including stochastic fuel-availability and managing the associated volatility by means of storage. The model is calibrated to the New Zealand electricity market and the market outcomes show that the management of storage materially affects them. The link between intertemporal allocation of generation and the mix of generation at any point in time is determined by the forward-looking shadow price of water. The price and quantity outcomes are volatile but much less volatile than is fuel availability and less volatility under monopoly than competition. The competitive market produces higher welfare in each trading period than the monopolist, with the monopolist economising relatively more than the competitive market on the use of gas. The monopolist runs the market less hard and thereby produces a less volatile sequence of market outcomes for the same reservoirs.

The model suggests that if climate change reduces long-run hydro fuel availability it will reduce welfare significantly; if it increases the volatility of fuel availability then it will also reduce welfare, but to a relatively small extent. Reduced predictability of inflows has a similar effect to that of increased variation in them. We show that an expansion in the reservoir would be of benefit where inflows become more volatile, but of negligible benefit when average inflows fall. In general there may be a conflict between consumers and generators in that in a number of circumstances reservoir expansion (holding inflow characteristics constant) reduces consumer surplus, raises producer surplus, and raises total surplus. This conflict is especially evident under competition, when price volatility is relatively high.

Our approach can be developed in a number of ways, although it will have tractability limitations as it requires a low number of state variables. An important development would be to make demand stochastic. This additional source of volatility and uncertainty is characteristic of electricity markets and would likely affect the management of storage and the value attached to storage facilities. Incorporating the features of an emissions trading scheme would also be fruitful. These are also likely to require incorporation of storage of a quantum (emission credits) with volatile prices. The market structure of monopoly is a polar case and work is proceeding on alternative oligopoly structures.

# Appendices

## A Calibration

The New Zealand spot market, in common with many other electricity markets, operates with dispatch determined by a uniform price auction. The auction results in a single price at each market node of the network.[25] We calibrate the model to the New Zealand electricity market.[26] Monthly data on New Zealand aggregate water inflows for the period July 1931–June 2008 imply the following stochastic process for $y_t$:

$$dy_t = 6.92(24.7 - y_t)dt + 4.49\sqrt{y_t}d\xi_t \tag{A-1}$$

which has unconditional mean and standard deviation of 24.7 TWh/y and 6.0 TWh/y, respectively.[27]

We assume that all non-hydro generation is gas generation[28] and calibrate the hydro and gas plant capacities as follows. In December 2007 the installed capacity of NZEM was 9396 MW, of which 5349 MW was hydro. If all generators ran at full capacity continuously for a year, hydro generation would be 47 TWh and non-hydro generation 35 TWh. These are used as the annual

[25] This is a characteristic of the institutional electricity markets of the CAISO, PJM, ERCOT and NYISO (see http://www.ferc.gov/industries/electric/indus-act/rto.asp, accessed 11 May 2010), and the Australian electricity market (NEM) (see http://epress.anu.edu.au/cs/mobile_devices/ch11s03.html, accessed 11 May 2010). The NZEM is described at http://www.electricity.commission.govt.nz (accessed 15 April 2010).

[26] All inflow, demand, and spot-price data were obtained from the New Zealand Electricity Commission's Centralised Dataset (http://www.electricitycommission.govt.nz/opdev/modelling/centraliseddata). Generation capacity figures are from the Ministry of Economic Development (2009).

[27] When we simulate daily data on inflows, we will use $y_{t+dt} = 0.4683 + 0.9810y_t + 0.23502\sqrt{y_t}N(0,1)$. The effect of this will be that in NZEM the shadow price of water—and hence generation decisions—at any date $t$ will reflect expectations of future inflows that are informed by the history of inflows.

[28] The quantum of generation that is not fossil-fuel fired or hydro was 13.5% in 2008 (Electricity Commission, 2009, p. 6). It largely consists of plants such as geothermal and wind that are must-run in nature. To some extent these are captured in the model by increasing marginal cost and the relatively low transmission cost of gas which means that some "gas" plants always run in the model. Coal produced 10.5% of generation in 2008, and this too can be assumed to be treated in the gas component of the model.

capacities of hydro and gas plants, respectively. We take the capacity of the lake to be that reported for storage capacity in June 2006, 4.44 TWh.

The subsequent analysis suggests that the price to consumers will be at least the marginal cost of supply. Taking the average daily price of electricity at the Haywards node in the 2007 calendar year (\$52.41/MWh) as the marginal cost of gas at its associated market clearing quantity of 18 TWh, we calibrate $b$ in the gas marginal cost function $c'(m) = 2bm$ to be $b = 1.46$.

The transmission cost parameters are estimated by price ratios between nodes taken to represent the generators and consumers.[29] Because the bulk of hydro generation is in the South Island the node taken for this generation plant is the southern, Benmore, node. Given that the largest consumer market is towards the north of the North Island, Otahuhu is assumed to be the consumer node, and the Haywards node lying between the two previously described nodes is taken as that for gas generation. Using daily average prices for the 2007 calendar year, we find that the average Benmore–Otahuhu relative price is $k_1 = 0.956$ and the average Haywards–Otahuhu relative price is $k_2 = 0.984$.

The consumer demand function is assumed to be linear with an elasticity of $-0.4$ at the average electricity consumption and price for the 2007 calendar year.[30] This elasticity was assumed by Borenstein and Bushnell (1999) in their study of the California electricity market. They used a constant elasticity, but linear demand enables the elasticity to vary with price–quantity pairs which seems a reasonable assumption where these vary significantly. The average price at the Otahuhu node was \$52.76/MWh and nationwide demand was 38 TWh. In combination with the elasticity, this yields the inverse demand curve $\varphi(q) = 185 - 3.47q$.

Table 6: Market outcomes with a $\$50/tCO_2$ carbon tax

| Quantity | Units | Mean | | Standard deviation | |
|---|---|---|---|---|---|
| | | Comp. | Monop. | Comp. | Monop. |
| Inflow ($y_t$) | TWh/y | 24.54 | 24.54 | 6.13 | 6.13 |
| Storage ($s_t$) | TWh | 1.42 | 2.04 | 1.28 | 1.49 |
| Price ($p_t$) | $/MWh | 58.61 | 97.51 | 6.50 | 2.84 |
| Hydro gen. ($z_t$) | TWh/y | 24.45 | 24.04 | 3.47 | 2.18 |
| Gas gen. ($m_t$) | TWh/y | 13.26 | 2.27 | 1.47 | 1.28 |
| MC of hydro ($h_t$) | $/MWh | 55.89 | 9.53 | 5.59 | 5.05 |
| MC of gas ($c'(m_t)$) | $/MWh | 57.67 | 9.86 | 6.39 | 5.59 |
| Welfare flow ($TS_t - c(m_t)$) | million $/y | 4043 | 3546 | 184 | 97 |
| Profit flow ($PS_t - c(m_t)$) | million $/y | 1736 | 2441 | 84 | 28 |
| Social value of 1TWh of capacity | million $ | 16 | 66 | | |
| Market value of 1TWh of capacity | million $ | 49 | 29 | | |

# B Additional results

# References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[30] The year 2007 was chosen as it was not a year of extreme inflows.

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